KTH Architecture and
the Built Environment

Frequency Optimization of Vibratory Rollers and Plates for Compaction of Granular Soil

Carl Wersäll

Doctoral Thesis
Department of Civil and Architectural Engineering
Division of Soil and Rock Mechanics
Royal Institute of Technology
Stockholm, 2016

TRITA-JOB PHD 1022
ISSN 1650-9501
ISBN 978-91-7729-042-1

Preface

This project was carried out between February 2011 and April 2016 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, KTH Royal Institute of Technology in Stockholm, Sweden. Supervisor was Prof. Stefan Larsson at KTH and assistant supervisor was Dr. Nils Rydén at the Faculty of Engineering, Lund University (LTH) and PEAB AB.

The project was funded by the Development Fund of the Swedish Construction Industry (SBUF), Dynapac Compaction Equipment AB, the Swedish Transport Administration, PEAB AB, Fredrik Bachman Memorial Fund and KTH Royal Institute of Technology. Sincere thanks go to the funders, making this project possible.

I would like to express my gratitude to my supervisors Prof. Stefan Larsson and Dr. Nils Rydén for their support, enthusiasm and guidance throughout the project. The laboratory work would not have been possible without the assistance of Dr. Kent Lindgren at the Marcus Wallenberg Laboratory for Sound and Vibration Research, KTH, manufacturing and lending test equipment, calibrating measurement systems and always being available to solve big and small problems. Dynapac provided test equipment, staff and their research facility for the full-scale tests. I especially thank Ingmar Nordfelt for all assistance in planning, executing and analyzing the tests and providing valuable insights coming from his tremendous experience, and Michael Knutsson for conducting the practical work. I am also grateful to their colleagues at Dynapac for fruitful discussions throughout the project.

Dr. Anders Bodare and Dr. Rainer Massarsch at Geo Risk and Vibration Scandinavia AB have provided valuable comments throughout the work. Their contributions are highly appreciated. Furthermore, I would like to express my sincere gratitude to all my colleagues at the Division of Soil and Rock Mechanics for valuable discussions and making my time at the department such an enjoyable experience. Finally, I would like to thank my family and my friends. Without their never-ending support, this work would not at all have been possible.

Stockholm, May 2016

Carl Wersäll

Abstract

Vibratory rollers are commonly used for compaction of embankments and landfills. This task is time consuming and constitutes a significant part of most large construction and infrastructure projects. By improving the compaction efficiency, the construction industry would reduce costs and environmental impact. In recent years, rollers have been significantly improved in regard to engine efficiency, control systems, safety and driver comfort. However, very little progress has been made in compaction effectiveness.

This research project studies the influence of the vibration frequency of the drum, which is normally a fixed roller property, and whether resonance can be utilized to improve the compaction efficiency. Frequency is essential in all dynamic systems but its influence on roller compaction has not before been studied. The concept of resonance compaction has previously been applied successfully in deep compaction of fills and natural deposits.

In order to examine the influence of vibration frequency on the compaction of granular soil, small-scale compaction tests of sand were conducted under varying conditions with a vertically oscillating plate. Subsequently, full-scale tests were conducted using a vibratory soil compaction roller and a test bed of crushed gravel. The results showed that resonance can be utilized in soil compaction by vibratory rollers and plates and that the optimum compaction frequency from an energy perspective is at, or slightly above, the coupled compactor-soil resonant frequency. Since rollers operate far above resonance, the compaction frequency can be significantly reduced, resulting in a considerable reduction in fuel consumption, environmental impact and machine wear.

The thesis also presents an iterative equivalent-linear method to calculate the frequency response of a vibrating foundation, such as a compacting plate or the drum of a roller. By incorporating strain-dependent soil properties, dynamic parameters such as acceleration and force could be predicted at large strain. The method seems promising for predicting the resonant frequency of the roller-soil system and can be used to determine the optimum compaction frequency without site- and roller-specific measurements.

Sammanfattning

Normalt används vibrationsväktar vid packning av fyllningar och väg- och järnvägsbankar. Denna aktivitet är tidskrävande och utgör en bytande del av de flesta stora bygg- och infrastrukturprojekt. Genom att effektivisera packningsprocessen skulle byggbranschen kunna minska kostnader och miljöpåverkan avsevärt. Under senare år har vältar utvecklats med avseende på motorer, kontrollsystem, säkerhet och förarkomfort. Dock har mycket liten utveckling skett av packningseffektiviteten.

I detta forskningsprojekt studeras inflytandet av valsens vibrationsfrekvens, vilken vanligtvis är en icke-variabel vältparameter, och huruvida resonans kan utnyttjas för att effektivisera packningsprocessen. Frekvensen är av avgörande betydelse i alla dynamiska system men dess inflytande på packningseffektiviteten för vibrationsvältar har inte tidigare studerats. Konceptet med resonanspackning har dock tillämpats inom djuppackning av fyllningar och naturliga avlagringar med goda resultat.

För att undersöka frekvensens betydelse vid packning av friktionsjord utfördes småskaleförsök under varierande förhållanden med en vertikalt vibrerande platta på en bädd av sand. Efter dessa utfördes fullskaleförsök med en vibrerande jordpackningsvält och krossat bärlägergrus. Resultaten visade att resonans kan utnyttjas vid jordpackning med vibrerande vältar och plattor och att den optimala packningsfrekvensen utifrån ett energiperspektiv är vid, eller något över, systemets resonansfrekvens. Eftersom vältar arbetar långt över resonans kan packningsfrekvensen sänkas avsevärt med en betydande minskning av bränsleförbrukning, miljöpåverkan och maskinslitage som följd.

Avhandlingen presenterar också en iterativ ekvivalentlinjär metod för att beräkna frekvensresponsen av ett vibrerande fundament, såsom en packningsplatta eller valsen på en vält. Genom att ta hänsyn till töjningsberoende jordegenskaper kan dynamiska parametrar, såsom acceleration och kraft, beräknas vid stora töjningar. Metoden verkar lovande för att förutsäga resonansfrekvensen av vält-jord-systemet och kan användas för att bestämma den optimala packningsfrekvensen utan plats- och maskinspecifika mätningar.

List of Publications

The following papers are appended to the thesis:

Paper I

Wersäll, C. and Larsson, S., 2013, "Small-Scale Testing of Frequency-Dependent Compaction of Sand Using a Vertically Vibrating Plate," *Geotechnical Testing Journal*, Vol. 36, No. 3, pp. 1-10.

Wersäll conducted the experiments, performed the analysis and wrote the paper. Larsson supervised the work and assisted in interpreting the results and writing the paper.

Paper II

Wersäll, C., Larsson, S., Rydén, N. and Nordfelt, I., 2015, "Frequency Variable Surface Compaction of Sand Using Rotating Mass Oscillators," *Geotechnical Testing Journal*, Vol. 38, No. 2, 2015, pp. 1–10.

Wersäll conducted the experiments, performed the analysis and wrote the paper. Larsson supervised the work and assisted in writing the paper. Rydén and Nordfelt assisted in interpreting the results and provided valuable comments on planning the tests and writing the paper.

Paper III

Wersäll, C., Larsson, S. and Bodare, A., 2014, "Dynamic Response of Vertically Oscillating Foundations at Large Strain," *Proceedings of the 14th International Conference of the International Association for Computer Methods and Advances in Geomechanics*, Kyoto, Japan, 22-25 September 2014, pp. 643-647.

Wersäll conducted the experiments, performed the analysis and wrote the paper. Bodare assisted in developing the analysis technique. Larsson supervised the work and assisted in writing the paper.

Paper IV

Wersäll, C., Nordfelt, I. and Larsson, S., "Soil Compaction by Vibratory Roller with Variable Frequency," Submitted to Géotechnique, February 2016.

Wersäll and Nordfelt planned, conducted and analyzed the tests. Wersäll wrote the paper. Larsson assisted in planning and analysis of the tests and in preparation of the manuscript.

Paper V

Wersäll, C. and Larsson, S., "Influence of Force Ratio and Frequency on Vibratory Surface Compaction," Submitted to Geotechnics for Sustainable Infrastructure Development – Geotec Hanoi 2016.

Wersäll conducted the experiments and performed the analysis. Larsson supervised the work and assisted in interpretation of measurements. The paper was written by Wersäll and Larsson.

Contents

List of Symbols

A	Dimensionless stiffness coefficient
B_z	Mass ratio
D_r	Relative density
F	Force
F_0	Force amplitude
F_c	Force in damper element
F_k	Force in spring element
F_m	Force in mass element
G	Shear modulus
G_{max}	Small-strain shear modulus
L_ε	Length of strained element
M	Dynamic magnification factor for constant force
M'	Dynamic magnification factor for rotating mass oscillators
N	Number of loading cycles
OCR	Overconsolidation ratio
P	Power
P_a	Atmospheric pressure
PI	Plasticity index
S	Settlement
W_c	Energy consumed in one cycle
a	Acceleration
a_0	Acceleration amplitude
c	Damping coefficient
c_{cr}	Critical damping coefficient
c_R	Rayleigh wave speed
c_S	Shear wave speed
e	Eccentricity
e	Void ratio
d_{10}	Diameter for 10 % passing
d_{60}	Diameter for 60 % passing
f	Frequency
f_r	Resonant frequency
g_{RMS}	RMS value of function g
k	Spring stiffness
k	Exponent depending on PI
m	Mass
m_0	Mass of foundation
m_e	Eccentric mass
$m_e e$	Eccentric moment

m_s	Apparent mass
n	Exponent depending on stress state
r_0	Footing radius
t	Time
u	Displacement
u_0	Displacement amplitude
u_A	Nominal displacement amplitude
u_{LVDT}	Settlement of the plate
u_{RMS}	RMS value of displacement
v	Velocity
v_0	Velocity amplitude
v_{LVDT}	Displacement velocity of the plate
w	Water content
ΔV_c	Compacted volume
ΔV_d	Displaced volume
α	Empirical factor depending on PI
β	Dimensionless frequency
β	Empirical exponent depending on PI
γ	Shear strain
γ_0	Shear strain amplitude
γ_r	Reference shear strain
ε	Compressive strain
ε_0	Compressive strain amplitude
ζ	Damping ratio
ζ_{max}	Maximum damping ratio
ν	Poisson's ratio
ρ	Mass density
$\sigma'_0{}^n$	Effective isotropic confining pressure
τ	Shear stress
ω	Circular frequency
ω_n	Circular natural frequency

1 INTRODUCTION

1.1 Background

Soil compaction is the most common ground improvement method and is often necessary to reduce settlement, increase stability and stiffness of the subgrade, control swelling and creep, lower the risk of liquefaction and decrease the permeability. It implies densification of the soil by reducing its pore volume. In granular soil, this is normally achieved by vibration or impact, producing stress-waves that rearrange the soil particles into a denser state. In construction of embankments and landfills, soil is placed in layers and compacted using vibratory rollers (Figure 1). This process is time-consuming and normally constitutes a significant part of the project cost as well as giving rise to considerable environmental impact. It is thus in the interest of the industry to improve the compaction efficiency and reduce the consumed time for this activity.

As vibratory rollers became popular around the 1950s, the optimum compaction procedure became a topic of research. One fundamental property that was investigated was the compaction frequency. All rollers operate with rotating eccentric mass oscillators that produce increasing force amplitude with frequency. However, all dynamic systems have a resonant frequency where vibrations are amplified. For roller compaction, this is within the operating frequency of the roller and taking advantage of this amplification can therefore be feasible. Several studies were conducted in the early years of this research field (especially in the 1950s and 1960s), with varying results. However, the available compaction equipment, measurement systems and evaluation techniques at the time were far from what they are today. Frequency was normally varied by adjusting the speed of the engine, which is a crude method for frequency variation. Since digital sensors or computers were not

Figure 1. Vibratory roller (courtesy of Dynapac Compaction Equipment AB).

available, results were difficult to interpret. Furthermore, there are many aspects that affect the results, not all of which were known at the time. First of all, a dynamic system behaves very differently below, close to or above resonance. Hence, it is important to be aware of the compaction frequency in relation to the resonant frequency. The acceleration amplitude is also of great importance. Several authors have found that compaction should be performed at accelerations above 1 g to be effective (D'Appolonia et al. 1969; Dobry & Whitman 1973). There are many other aspects, such as dynamic-to-static load ratio, shape of the contact surface and soil properties. Due to the complexity of the problem, the early studies had varying conclusions.

The first to propose a compactor utilizing frequency to obtain the maximum degree of compaction was Hertwig (1936). Tschebotarioff & McAlpin (1947) concluded that the subsidence of a piston, vibrating on the soil was independent of frequency as long as the total number of cycles was constant. However, the frequency in those tests was very low, less than 20 Hz. Bernhard (1952) conducted laboratory tests with variable frequency and constant force, obtaining a more efficient compaction at the resonant frequency. Converse (1953) conducted field compaction tests of sand and also concluded that resonance could be utilized. Forssblad (1965) highlighted that in the tests by Bernhard and Converse the dynamic load was only in the same order of magnitude as the static weight and argued that the results could not be compared to roller compaction. Several other authors found a correlation between resonance and increased compaction efficiency (Johnson & Sallberg 1960; Lorenz 1960). Forssblad (1965) argued that the increase in force amplitude with frequency would be too significant for the resonant amplification to influence the compaction effect and that the technical difficulties for utilizing resonance would exceed the practical advantages. Thus, there was no agreement among researchers on whether resonant amplification could be used to improve roller compaction. There was, however, one conclusion on with the community agreed, namely that effective compaction must be performed above the resonant frequency.

There have been many attempts to model the roller behavior by mathematical or numerical methods. Yoo & Selig (1979) presented a lumped-parameter model that formed the basis for most subsequent models of roller behavior. These studies have mainly been conducted for the purpose of continuous compaction control and intelligent compaction (e.g., Forssblad 1980; Thurner & Sandström 1980; Adam 1996; Anderegg & Kaufmann 2004; Mooney & Rinehart 2009; Facas et al. 2011). Modeling the dynamic behavior of compaction equipment is complicated by the fact that soil shows very nonlinear stress-strain behavior. Most models do not take this into account. However, Susante & Mooney (2008) developed a model that includes nonlinear soil stress-strain behavior, calculating the response in time domain.

In the 1970s, computer programs using an equivalent-linear approach to determine the nonlinear seismic response during earthquakes, such as SHAKE (Schnabel et al. 1972), became popular. These programs apply an iterative procedure to determine the nonlinear response of a transient time history. As finite element and other numerical methods were introduced, these became dominating in calculating the nonlinear response. However, numerical methods are time-consuming and require skilled operators to be reliable. Thus, equivalent-linear methods are still useful but there has hardly been any development of these concepts in recent years. No one has previously used this approach for calculating the nonlinear response of an oscillating foundation (such as surface compaction equipment) on soil with strain-dependent properties.

1.2 Objectives

This research project aims at determining the optimum compaction frequency of vibratory rollers and surface compaction plates and to investigate whether resonance in the coupled compactor-soil system can be utilized for increasing the compaction efficiency. First, the fundamental dynamic behavior during frequency-variable compaction is studied in small-scale tests. These are conducted under varying conditions to quantify the effect of, not only frequency, but also type of oscillator, dynamic load and soil water content. The small-scale tests form a basis for full-scale tests using a vibratory roller, where gravel is compacted at various frequencies to study the effect on compaction efficiency.

Another objective is to develop an equivalent-linear calculation procedure that can be performed in frequency domain to predict the resonant frequency and the dynamic response of vibratory rollers and plates. These calculations are compared to results of the small-scale and full-scale tests.

1.3 Outline of Thesis

This thesis consists of an introductory part and five appended papers, two published in peer-reviewed journals, one submitted journal article, one published and presented at an international conference and one submitted conference paper. The introductory part is intended as an introduction and a complement to the appended papers. It contains background information, summary of the main findings and further development of some concepts that are included in the papers.

Chapter 2 is a description of the small-scale tests. Since the tests are described thoroughly in the papers, this chapter only summarizes briefly the test setups and provides additional photographs of the equipment. The test results of the two journal papers on the small-scale tests are summarized and discussed in relation to each other.

Chapter 3 summarizes the full-scale tests and presents additional results that are not included in the paper.

Chapter 4 describes the fundamentals of dynamic single degree of freedom systems and vertically oscillating foundations. The equivalent-linear calculation procedure developed in Paper II is described in detail.

Chapter 5 contains a summary of the appended papers.

Chapter 6 provides the main conclusions of the thesis.

2 SMALL-SCALE TESTS

Small-scale compaction tests were conducted in laboratory environment. The tests were divided over two main setups and several test series. An electro-dynamic oscillator was used in the first setup, creating high controllability. The second setup utilized rotating mass oscillators with less controllability but higher resemblance to field conditions. Both setups were purpose-built for the tests. Since the tests are described in detail in the appended papers, this chapter provides only a summary of the test setups and results.

Sand was placed in a box having inner measurements 1100 mm x 700 mm x 370 mm (width x length x height). The boundaries were coated with 30 mm of expanded polystyrene to reduce vibration reflections and the bottom of the box consisted of the concrete floor below the box. The filling method is crucial to obtain similar test conditions as it has a strong influence on the initial density of the sand (Rad & Tumay 1987). Due to the large number of tests and the large sand volume, the material was filled by pouring. Since there was no target density but rather a similar density in all tests that was important, this method was considered sufficient. Other more precise methods, such as raining, would be unrealistically time-consuming. The pouring was performed in the same way by the same person to minimize any differences in initial density.

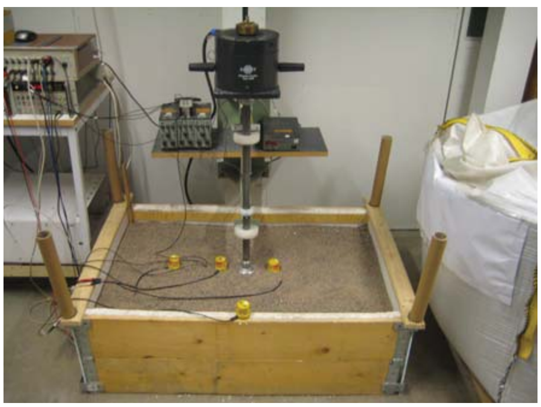
2.1 Tests with Electro-Dynamic Oscillator

This type of oscillator consists of a static mass and a significantly smaller oscillating mass on top. In the first setup, shown in Figure 2, the static mass was connected to a steel rod with a circular steel plate, 84 mm in diameter, at the other end. The rod was running through two low-friction polytetrafluoroethylene (Teflon) rings, allowing the rod to move only in the vertical direction. The plate was placed directly on the sand surface. The advantage with an electro-dynamic oscillator is that dynamic quantities can be adjusted in real-time, thus making the tests very controllable. The system can be illustrated as a 2DOF coupled mass-spring-dashpot model where the dynamic force is generated in the spring between the oscillating and static masses. Since the oscillating mass is much smaller than the static mass, the soil response does not influence its vibrations, which means that measurements on the oscillating mass are independent of soil-compactor resonances. This provides the opportunity to conduct tests under constant dynamic load. The total mass of the vertically moving system was 37.4 kg.

One accelerometer was placed on the oscillating mass and one on the static mass. A force transducer was placed between the plate and the rod measuring the reaction force. Furthermore, the rod was connected to a linear variable differential transformer (LVDT), measuring the vertical settlement of the plate. Acceleration signals from the accelerometers were integrated in the amplifiers so that particle velocity was recorded. An external amplifier controlled the amplitude of the oscillator and the frequency was adjusted by a function generator. Geophones were placed in the sand, on the box perimeter and on the concrete floor. A vertical accelerometer was buried in the sand, 20 cm below the plate.

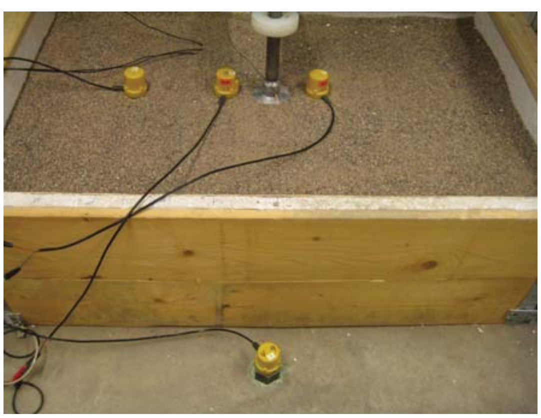
(a)

(b)



(c)

(d)



(e)

(f)

Figure 2. Tests with electro-dynamic oscillator. (a) Preparation of test box. (b) Preloading. (c) The complete test setup. (d) Settlement and heave after compaction. (e) Measurement with geophones inside and outside of the test box. (f) Test for investigation of soil displacement.

Each test was conducted with frequency sweep and a constant particle velocity on the moving mass. The frequency was controlled by the function generator and the velocity amplitude was adjusted manually on the oscillator amplifier. The measured acceleration signal was integrated and plotted on a computer screen in real-time for adjusting the amplitude. The tests are described thoroughly in Paper I.

2.2 Tests with Rotating Mass Oscillators

To obtain conditions that are more similar to those during roller compaction, a new compactor was manufactured using two rotating mass oscillators, together giving rise only to a vertical component. Except for the new type of oscillators, the equipment, shown in Figure 3, was very similar as in the previous small-scale tests. A mass-spring-dashpot representation would here only include one mass where the force is directly applied. An accelerometer was mounted on the bottom plate and a force transducer was placed between the plate and the rod. In the same manner as the previous tests, the vertical settlement was measured by an LVDT. In some tests, geophones were placed in the sand and on the box perimeter or outside the box. The mass of the original vertically moving system was 28.8 kg, which was then reduced to 22.1 kg and 13.0 kg to investigate the influence of the static weight.

The tests, described in Paper II and Paper V, were conducted at discrete frequencies, i.e. not using frequency sweep as in the previous tests. The sand was replaced between each test. When using rotating mass oscillators, the eccentric moment is constant and the applied force increases with the square of frequency. It was thus not possible to control particle velocity or any other dynamic quantity. This is true also for compaction with vibratory roller. In each test, the sand was compacted for 30 seconds and the settlement was recorded.

2.3 Results of Small-Scale Tests

In the tests using the electro-dynamic oscillator, compaction was significantly enhanced close to the resonant frequency with hardly any compaction sufficiently below or above this frequency. The tests with rotating mass oscillators, on the other hand, showed a more complex relationship between frequency and compaction. The applied force increased with frequency, producing a very high degree of compaction at the higher frequencies and hardly any compaction at the low frequencies. In the mid-range, however, there was a resonant amplification, which was quite modest compared to the amplification in the previous tests. The main differences between the two test setups were the following:

- Higher dynamic loads in the second setup.
- The force ratio, i.e. ratio of dynamic load and static weight.
- The variation of input load with frequency – constant particle velocity in the first test setup and force increasing proportional to the square of frequency in the second setup.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 3. Tests with rotating mass oscillators. (a) Test setup. (b) Oscillators with protective caps removed. (c) Preloading by vibrating a wooden plate. (d) After completion of a test on dry sand. (e) After completion of a test on wet sand. (f) Imprint in wet sand after test and removal of the plate.

The difference in resonant amplification originates from the differences between tests, as listed above. Since the dynamic-to-static force ratios are significantly lower than one and far above one, respectively, the fundamental dynamic behavior is essentially different. However, a slight modification of the force ratio does not influence the results significantly, as is shown in Paper V. Since force is increasing drastically with frequency during operation of the rotating mass oscillators, resonant amplification becomes less pronounced. The most influential aspect, however, is most likely the high dynamic load, giving rise to large strains, which produces a significant reduction in the soil stiffness while the damping ratio increases, which will be explained in Chapter 4. This causes the amplification to reduce.

In the tests with rotating mass oscillators, the loading properties were similar to those of vibratory rollers. The results, presented in Paper II, suggested that the compaction efficiency is nearly constant in a quite wide frequency band above the resonant frequency. Rollers operating at the high end of this band could thus lower their compaction frequency without loss of efficiency. The reduction would imply a significant reduction in energy consumption. This hypothesis was the basis for the full-scale tests described in the next chapter.

3 FULL-SCALE TESTS

To determine the applicability of the results from the small-scale tests in practice, full-scale tests were conducted using a vibratory roller. The tests are described in detail in Paper IV. It was found in the small-scale tests that the optimum compaction frequency from an energy perspective was around the coupled compactor-soil resonant frequency. It was hypothesized that the same is true for a vibratory roller since the loading characteristics is essentially similar. However, due to continuous movement of the roller, resulting in several centimeters between impacts, and the difference between a plate and the drum of a roller when it comes to contact area and stress distribution, it was uncertain whether the small-scale test results were applicable in roller compaction. Furthermore, the compaction frequency in relation to the resonant frequency was not known. Mooney and Rinehart (2007) found the resonant frequency at a particular site to vary between 15 and 27 Hz with a smaller tandem roller. The roller used in the tests described herein had standard operating frequency of 31 Hz at high amplitude but was modified to operate in the range 15-35 Hz.

3.1 Test Description

Compaction was performed in six passes using a CA3500D single drum soil compaction roller with a weight of 12100 kg. Nine tests were conducted with a fixed operating frequency between 15 and 35 Hz. The following procedure was carried out at each frequency:

1. Loosening of the soil down to 0.6 m depth using an excavator.
2. One static pass over the entire test bed with a roller speed of 0.35 m/s.
3. Two vibrated passes on the test surface using high amplitude, a vibration frequency of 28 Hz and roller speed of 1 m/s.
4. Levelling in 60 points.
5. Nuclear density gauge measurements at three locations.
6. Two vibrated passes at the fixed frequency of that particular test (pass 1-2). Roller speed 1 m/s.
7. Levelling in 60 points.
8. Two more vibrated passes (pass 3-4).
9. Levelling in 60 points
10. Final two vibrated passes (pass 5-6).
11. Levelling in 60 points.
12. Nuclear density gauge measurements at three locations.

One sweep test was conducted where the frequency was varied linearly between 15 and 35 Hz in 12 passes without any preparatory compaction. In this test, only measurements on the roller were conducted. Execution of the full-scale tests is shown in Figure 4.

(a)

(b)

(c)

(d)

Figure 4. Full-scale tests. (a) Loosening of gravel. (b) Test bed and levelling points. (c) Test execution. (d) Measurement by nuclear density gauge.

3.2 Results of Full-Scale Tests

Three examples of vertical (z) and horizontal/longitudinal (x) acceleration time histories are shown in Figure 5, corresponding to frequencies below, at and above resonance, respectively. During the first approximately 10 seconds, the frequency is stabilizing while roller has not yet started moving along the test surface. The middle of the signal corresponds to when the roller stops and changes direction. To obtain a representative measure of the acceleration of one pass, the RMS value is taken over the whole pass while the roller is within the test surface. Figure 6a shows the horizontal RMS acceleration as a function of frequency, which increases almost linearly and does not change with the number of passes. The vertical acceleration is shown in Figure 6b. Here, the amplitude is affected by the response of the soil, resulting in a modest amplification at the resonant frequency, 17 Hz. The acceleration is also generally higher after a larger number of passes due to increased stiffness of the soil. The dynamic displacement is obtained by double integration of the acceleration record. The RMS value of vertical displacement, presented in Figure 6c, shows a significant amplification at the resonant frequency and a large difference between different passes. Higher number of passes and high frequencies yields amplification due to double jump of the drum.

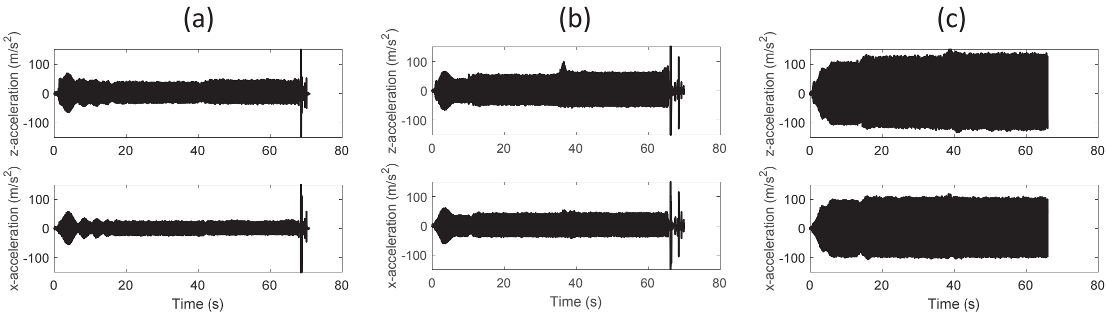


Figure 5. Examples of vertical and horizontal acceleration on the drum, (a) below resonance, (b) at resonance and (c) above resonance.

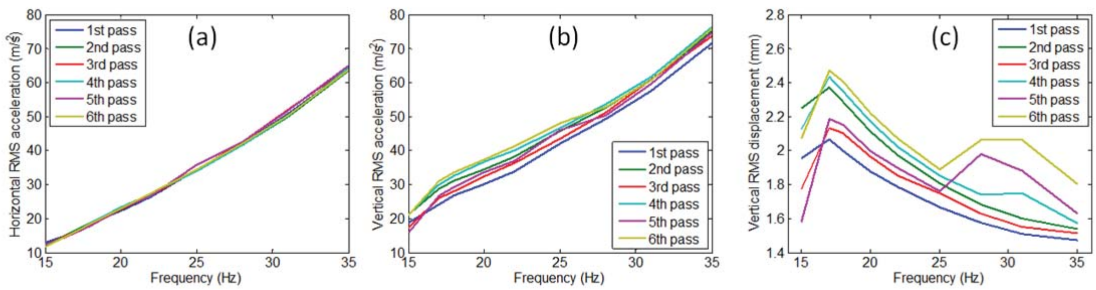


Figure 6. Calculated RMS values, as a function of frequency, of (a) horizontal acceleration, (b) vertical acceleration and (c) vertical displacement.

All settlement records are shown in Figure 7, where each data point is the average of the three levelling measurements along the width of the drum. In some of the records, the end points show a deviating settlement. Thus, the end points have been omitted in calculation of the average settlement, shown in Figure 8. The frequency-dependent settlement suggests that the optimal compaction frequency is slightly above the resonant frequency. Although the resonant amplification is quite modest, the test results indicate that the compaction frequency can be lowered significantly without any loss in compaction efficiency. The particular roller used in this test has a standard operating frequency of 31 Hz at high amplitude while the optimal frequency is around 18 Hz. Such a reduction in frequency yields a significant reduction in fuel consumption and environmental impact while increasing the total life span of the machine.

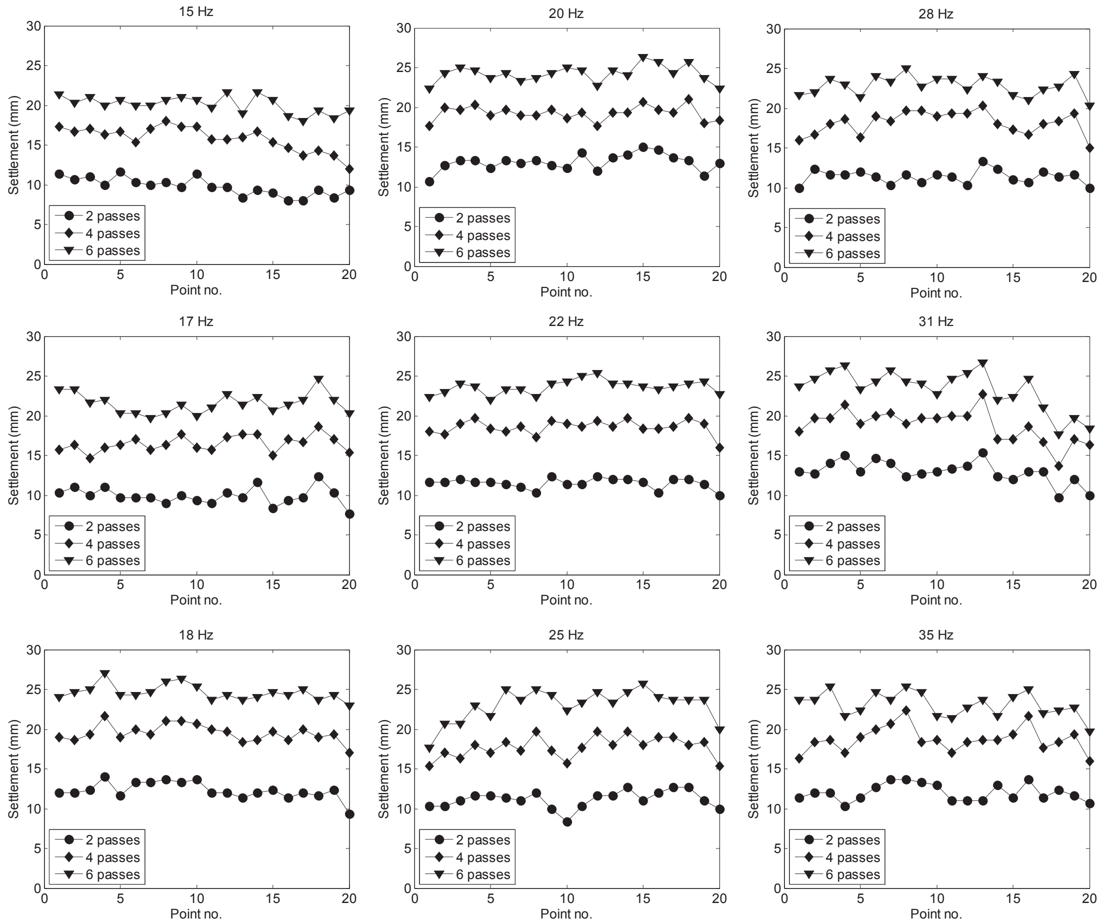


Figure 7. Settlement along the test surface for all frequencies. Each data point represents the average of three measuring points.

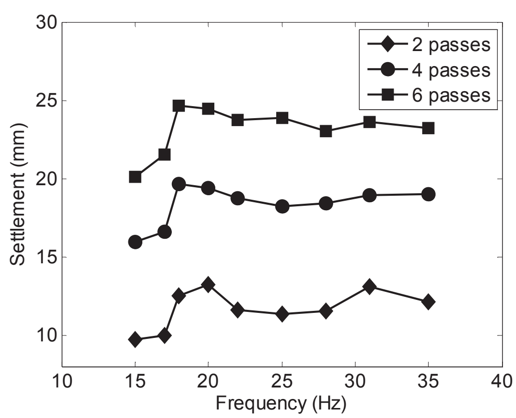


Figure 8. Average settlement.

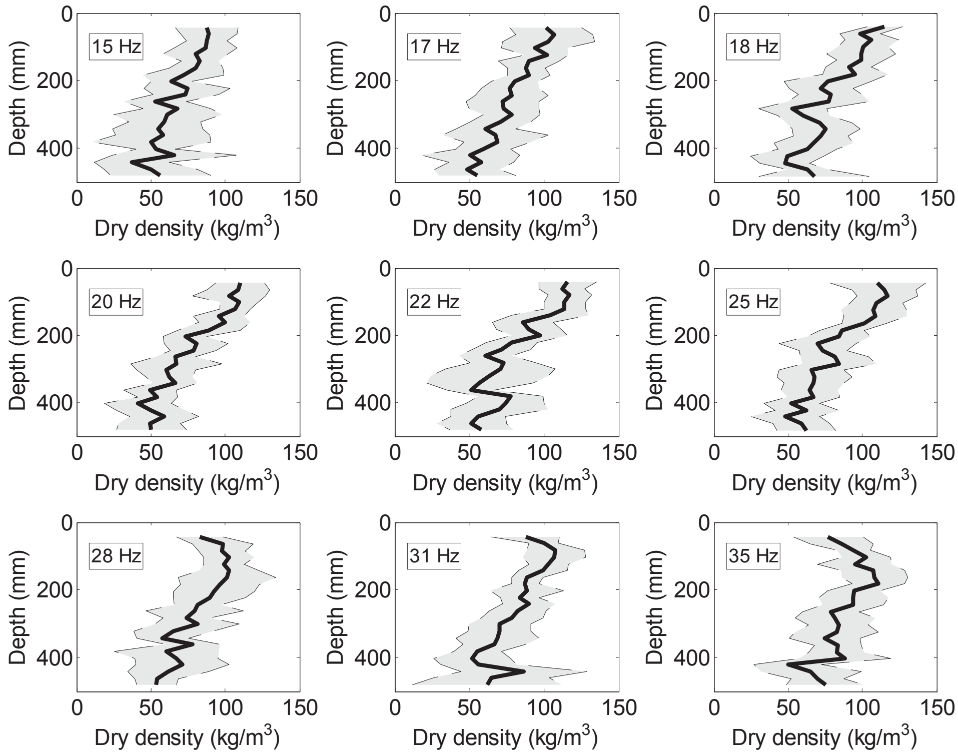


Figure 9. Density increase from six passes for each frequency.

Depth-dependent density increase was determined before and after the six main passes by nuclear density gauge. Figure 9 shows the increase in density for each frequency with its corresponding sample standard deviation. It is obvious that the increase is not constant with depth but rather greater close to the ground surface and thus the drum of the roller. It can also be seen that the top layer of soil is loosened at higher frequencies. The depth and frequency dependence and its implications are discussed in Paper IV. The density measurements also illustrate the variation in initial density, i.e. after preparatory compaction. Figure 10 shows the average initial density and that after six passes. It is observed that the initial density variation is greater than the increase obtained in the six main passes, illustrating the small increase in the parameter resulting from the tests. This is due to the fact that the relative density is already close to its maximum and the settlement is thus considered a more reliable parameter for average densification. Furthermore, the initially denser material at 17 Hz suggests that the settlement would have been greater at that frequency if the initial density had been constant for all frequency. In a similar manner, the settlement would have been smaller at 25 Hz and 28 Hz, enhancing the frequency-dependent settlement behavior.

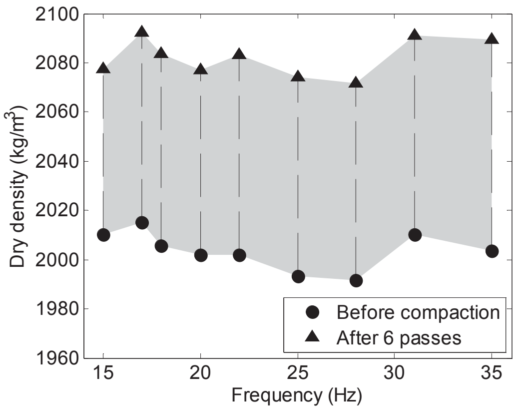


Figure 10. Average density before and after compaction.

4 PREDICTING THE DYNAMIC RESPONSE

This chapter describes how to predict the dynamic response of oscillating foundations, such as compaction equipment, by an equivalent-linear approach. Due to a high degree of soil nonlinearity, elastic formulations are not sufficient for analyzing dynamic parameters at high strain. For example, the method described herein can be used for predicting the resonant frequency of a roller-soil system and thus determine the optimum compaction frequency.

4.1 Oscillating Foundations on Softening Soil

Studies on vibrating foundations began with the objective to analyze ground vibrations from rotating machinery founded on the ground surface. This has become the basis for dynamic soil-structure interaction analysis, including many more applications than just rotating machinery, such as wind turbines and bridge abutments subjected to traffic load. This section describes how basic equations for vibrating foundations can be combined with empirical knowledge for nonlinear stress-strain behavior of soil to predict the dynamic response of vibrating foundations on softening soil. Since this thesis deals with vertical oscillations on granular soil, other oscillatory motions or plastic soils are not treated herein. For other vibration modes, such as rocking or horizontal oscillation, reference is made to Richart et al. (1970) and Gazetas (1983).

4.1.1 Single Degree of Freedom Systems

The dynamic behavior of a vertically oscillating foundation can be estimated by analyzing a single degree of freedom (SDOF) system consisting of a mass, a dashpot and a spring, where the force in these three components are proportional to acceleration, velocity and displacement, respectively. The forces in each element (F_m, F_c and F_k) are determined by Equations 1 to 3.

$$F_m = ma \tag{1}$$

$$F_c = cv \tag{2}$$

$$F_k = ku \tag{3}$$

where m is mass, a is acceleration, c is damping coefficient, v is vibration velocity, k is spring stiffness and u is displacement. Since the velocity is given by $v = \frac{du}{dt}$ and the acceleration is given by $a = \frac{d^2u}{dt^2}$, and since all forces need to be in equilibrium, a SDOF system can be described by the second order differential equation presented in Equation 4.

$$m\frac{d^2u}{dt^2} + c\frac{du}{dt} + ku = F(t) \tag{4}$$

where $F(t)$ is the externally applied force. The system may be under-damped, critically damped or over-damped depending on if the damping coefficient is less than, equal to or larger than the critical damping coefficient c_{cr}. The ratio between these is denoted damping ratio, ζ, and is shown in Equation 5.

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}} \tag{5}$$

All dynamic systems have one or several natural frequencies. The circular fundamental natural frequency, ω_n, of a SDOF system is calculated by Equation 6.

$$\omega_n = \sqrt{\frac{k}{m}} \tag{6}$$

It is convenient to express frequency normalized by the natural frequency, the so-called dimensionless frequency, β, as shown in Equation 7.

$$\beta = \frac{\omega}{\omega_n} \tag{7}$$

where ω is the circular frequency. For a harmonic external load, the solution to Equation 4 can be expressed by Equation 8.

$$u_0 = \frac{F_0}{k} \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} \tag{8}$$

where u_0 is displacement amplitude and F_0 is force amplitude. The dynamic displacement in relation to the displacement that would be obtained from static loading by the same force is called dynamic magnification factor. It is calculated by Equation 9 and shown in Figure 11 for different values of the damping ratio.

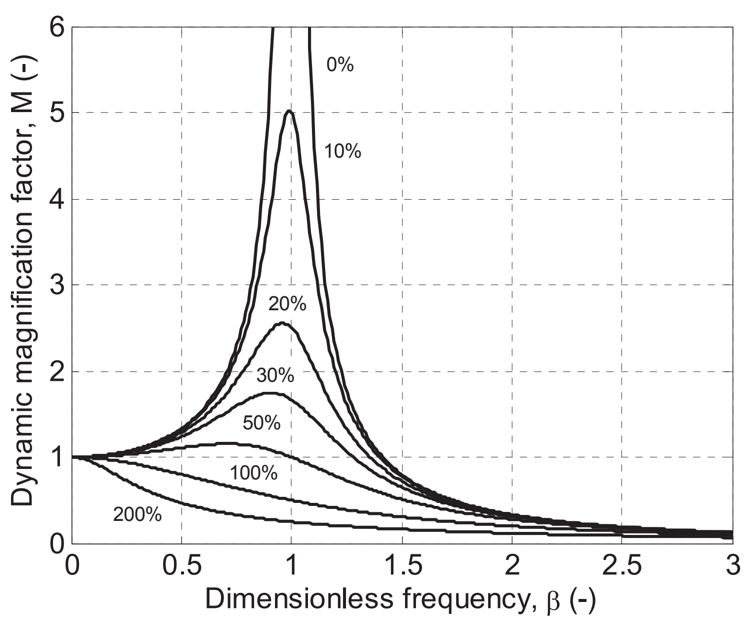


Figure 11. Dynamic magnification factor for constant force and different damping ratios.

$$M = \frac{u_0}{F_0/_k} = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} \tag{9}$$

When the frequency approaches zero, the magnification factor approaches unity. As the excitation frequency approaches the natural frequency, the dynamic response is significantly magnified. The frequency where maximum magnification occurs is the resonant frequency, which is equal to the natural frequency when damping is zero and slightly lower as the damping ratio becomes larger. When the frequency is increased above resonance, the magnification factor (and thus the displacement amplitude) decreases and approaches zero for large frequencies. If the damping ratio is zero, the resonant amplification is infinite. This is an unrealistic case as there are no real systems without damping. However, for a damping ratio of 10 %, which represents quite high damping, the resonant amplification is still as high as 5 times the static value. The damping ratios of 20-30 % shown in the figure are uncommon but can occur for example during large strain in soil, as will be discussed below.

The magnification factor shown in Figure 11 is for the case where the applied force amplitude is constant with frequency. If the load would be produced by rotating mass oscillators, force amplitude would increase rapidly with frequency according to Equation 10.

$$F_0 = m_e e \omega^2 \tag{10}$$

where m_e is the eccentric mass and e is the eccentricity. The nominal displacement amplitude of a rotating mass oscillator, u_A, is given by Equation 11.

$$u_A = \frac{m_e e}{m} \tag{11}$$

Details regarding the properties of rotating mass oscillators can be found in, for example, Forssblad (1981). By applying Equations 6 and 10 and 11, Equation 8 may be rewritten as Equation 12 with a corresponding dynamic magnification factor for rotating mass oscillators, *M*', given by Equation 13.

$$u_0 = u_A \frac{\beta^2}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} \tag{12}$$

$$M' = \frac{u_0}{u_A} = \frac{\beta^2}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} \tag{13}$$

The magnification factor is shown in Figure 12. Since the dynamic force is generated by the rotating masses, there is no or very little dynamic displacement when the frequency is zero or close to zero. Thus the magnification factor approaches zero when the frequency goes toward zero. The behavior around resonance is similar to the case with constant force, except from the resonant frequency being slightly larger than the natural frequency. As the frequency is increased, the magnification factor converges toward unity for all damping ratios, i.e. the displacement amplitude approaches the nominal amplitude. The curves in Figure 11 and Figure 12 are called frequency response functions since they describe the response of a system to frequency-dependent dynamic input variable.

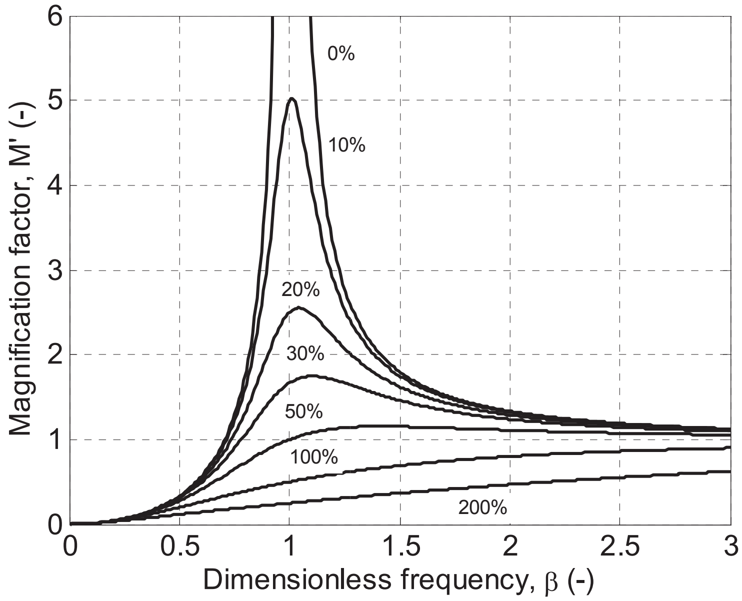


Figure 12. Dynamic magnification factor for rotating mass oscillators and different damping ratios.

Instead of magnification factor, they can be displayed for any other dynamic amplitude quantity, such as dynamic displacement, velocity, acceleration or force but are then not, by definition, frequency response functions. The displayed dynamic output is then herein simply denoted frequency response or response diagram.

4.1.2 Vertically Oscillating Foundations

The previous section described fundamental dynamic properties of SDOF systems. This section focuses on calculation of the dynamic response of a vertically oscillating foundation on an elastic half-space, as described by Lysmer & Richart (1966). For horizontal or rocking motion, see Hall (1967), and for torsion, see Richart et al. (1970). Gazetas (1983) presented equations for foundations on layered soil. Model tests have been conducted to experimentally determine the response of oscillating foundations under various conditions (e.g. Novak 1970; Baidya & Murali Krishna 2001; Mandal et al. 2012).

Lysmer & Richart (1966) showed how the behavior a vertically oscillating foundation on an elastic half-space can be simulated by a SDOF model, where spring stiffness and damping ratio are given by Equations 14 and 15.

$$k = \frac{4Gr_0}{1-\upsilon} \tag{14}$$

$$\zeta = \frac{0.425}{\sqrt{B_z}} \tag{15}$$

where G is soil shear modulus, r_0 is the footing radius, ν is Poisson's ratio of the soil and B_z is the mass ratio obtained by Equation 16.

$$B_z = \frac{1-\nu}{4}\frac{m}{\rho r_0^3} \tag{16}$$

where m is the total mass and ρ is the mass density of the soil. The total mass consists of two components. One is the mass of the foundation, m_0, including any external static load on it. The other part is called apparent mass, m_s, which corrects for the fact that stiffness decreases with frequency (Gazetas 1983). Different equations exist for calculating the apparent mass. In this study it becomes very small and is thus neglected. The total mass is given by Equation 17 and one expression for the apparent mass is given by Equation 18.

$$m = m_0 + m_s \tag{17}$$

$$m_s = \frac{1.08}{1-\nu}\rho r_0^3 \tag{18}$$

By applying the above equations to the SDOF model presented in the previous section, the dynamic behavior of a vertically oscillating foundation can be estimated. The main limitation with this and many other studies on the subject is the assumption that the subgrade is elastic. Since the stress-strain behavior of soil (especially non-plastic soil) is highly nonlinear, this simplification can lead to very large discrepancies between calculated and real dynamic responses. Soils with high plasticity, however, behave more elastic and the implications of treating the soil as perfectly linear are thus less severe. Nonlinear stress-strain behavior of soil, and a method to take these properties into account, is explained below.

4.1.3 Soil Nonlinearity

Deformation behavior of granular soil is often modeled by a hyperbolic stress-strain formulation (Kondner 1963a; Kondner 1963b; Hardin & Drnevic 1972a; Hardin & Drnevic 1972b). The hyperbolic shear stress τ is given by Equation 19.

$$\tau = \frac{G_{max}\gamma}{1+\left|\frac{\gamma}{\gamma_r}\right|} \tag{19}$$

where G_{max} is the small-strain shear modulus, γ is the shear strain and γ_r is a reference strain. At very low strains the shear modulus is at its maximum, hence the denotation G_{max}. Equation 19 describes the so-called backbone curve (also called virgin curve or skeleton curve), which applies to virgin loading. When soil is subjected to cyclic loading, the stress-strain relationship forms a hysteresis loop often modeled by Masing Rule (Masing 1926), which implies magnifying the backbone curve by a factor of two during unloading and reloading. The backbone curve and hysteresis loop are shown in Figure 13.

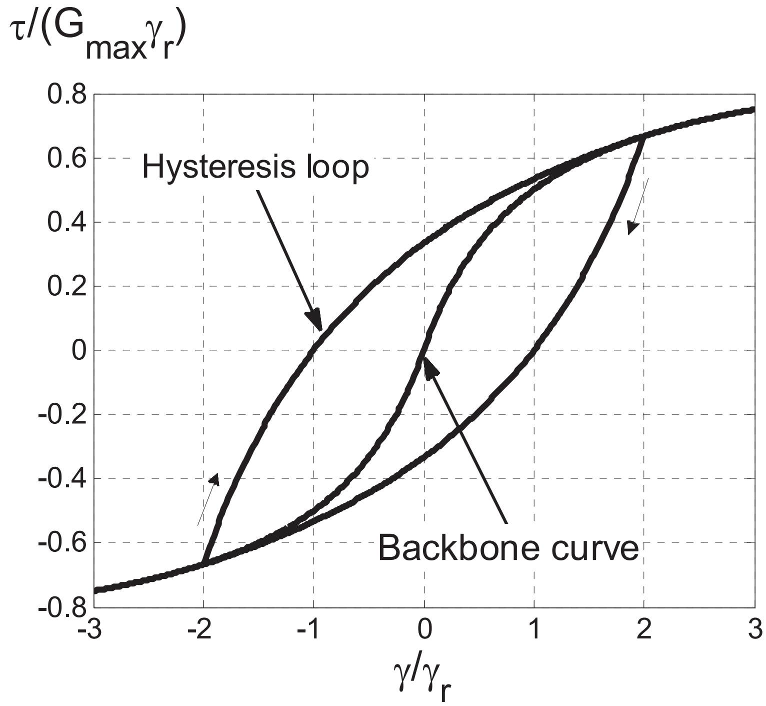


Figure 13. Normalized hyperbolic stress-strain relationship.

The small strain shear modulus can be estimated by Equation 20 according to Hardin (1978).

$$G_{max} = \frac{A \cdot OCR^k}{0.3 + 0.7e^2} P_a^{(1-n)} \sigma'^n_0 \tag{20}$$

where A and n are a dimensionless parameters, OCR is the overconsolidation ratio, k is a parameter depending on plasticity index (PI), e is the void ratio, P_a is the atmospheric pressure (100 kPa) and σ'^n_0 is the effective isotropic confining pressure. Studies investigating the values of A and n (e.g. Stokoe et al. 1999) have found A to vary in wide interval and n to vary slightly.

As can be seen in Figure 13, stiffness decreases with strain. Many authors have studied the strain-softening effect on the shear modulus (Seed et al. 1986; Vucetic & Dobry 1991; Rollins et al. 1998; Stokoe et al. 1999; Assimaki et al. 2000; Kausel & Assimaki 2002; Tatsuoka et al. 2003; Massarsch 2004; Zhang et al. 2005; Amir-Faryar et al. 2016, among others). In the hyperbolic formulation described above, the shear modulus G decreases according to Equation 21.

$$\frac{G}{G_{max}} = \frac{1}{1 + \left|\frac{\gamma}{\gamma_r}\right|} \tag{21}$$

The reference strain is a curve-fitting parameter that depends on soil properties. It represents the strain at which the shear modulus has half the value of the small strain shear modulus. As shear strain increases, it affects not only the shear modulus, but also the damping ratio increases significantly. Hardin & Drnevic (1972b) proposed the formulation for damping ratio presented in Equation 22.

$$\frac{\zeta}{\zeta_{max}} = 1 - \frac{G}{G_{max}} \tag{22}$$

where ζ_{max} is the maximum damping ratio, which depends on the soil type and number of loading cycles N. Equation 23 shows the maximum damping ratio (in percent) for clean dry sand and Equation 24 presents the same parameter for saturated sand.

$$\zeta_{max} = 33 - 1.5\log(N) \tag{23}$$

$$\zeta_{max} = 28 - 1.5\log(N) \tag{24}$$

Rollins et al. proposed a model for shear modulus, Equation 25, and damping ratio, Equation 26, based on tests conducted on gravel.

$$\frac{G}{G_{max}} = \frac{1}{1.2 + 16|\gamma|(1 + 10^{-20|\gamma|})} \tag{25}$$

$$\zeta = 0.8 + 18(1 + 0.15|\gamma|^{-0.9})^{-0.75} \tag{26}$$

There is an apparent uncertainty in Rollins' equations. The shear modulus reduction ratio does not approach one for small strains due to the factor 1.2 in the denominator. As the ratio has to become unity for zero strain, the most obvious assumption is that this is a misprint in the paper.

A further formulation for shear modulus was proposed by Massarsch (2004), as presented in Equation 27.

$$\frac{G}{G_{max}} = \frac{1}{1 + \alpha|\gamma|(1 + 10^{-\beta|\gamma|})} \tag{27}$$

where α and β are empirical factors depending on PI. The variation of α and β are shown in Figure 14. The study was conducted with focus on fine-grained soils and thus no values are available for PI less than 10 %. However, Stokoe et al. (1999) found that strain-softening relationships of natural non-plastic soils and soils with low plasticity are very similar. The behavior of granular soil (non-plastic) can thus be estimated by applying values for PI = 10 %.

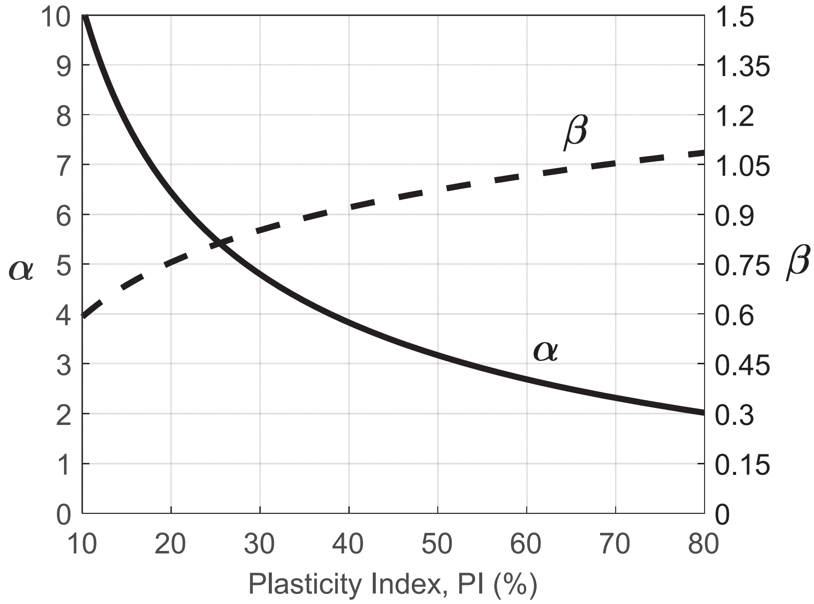


Figure 14. Variation of α and β with PI (after Massarsch 2004).

The shear modulus reduction ratios according to Equations 21, 25 and 27 are shown in Figure 15. A reference strain of 0.05 % was chosen, which is a typical value for sand (Stokoe et al. 1999). A modified version of Rollins' equation is also shown, where the term 1.2 has been replaced by 1.0. The reduction according to Massarsch is very similar to Hardin & Drnevic at low strains but implies slightly higher values of the shear modulus at large strains. The original expression by Rollins et al. is obviously not correct at small strains. The modified equation shows smaller shear modulus than the other expression at small and moderate strains, while larger at quite high strain level and similar to the other curves at very high strains.

Figure 16 shows the damping ratio calculated by Equation 26 (Rollins et al.), Equations 22 and 23 (Hardin & Drnevic, dry sand, first loading cycle) and Equations 22 and 24 (Hardin & Drnevic, saturated sand, first loading cycle). Seed et al. (1986) compiled results from many laboratory and field studies for strain-dependent damping ratio of sand. Equation 22 (using the reference strain 0.05 %, as above) is fitted to that data and shown in the same figure. The data from Seed et al. show a damping ratio close to that of clean saturated sand according to Hardin & Drnevic. For clean dry sand, the damping ratio is higher. The curve from Rollins et al. has a significantly lower damping ratio at high strains. All curves based on hyperbolic strain have one major disadvantage, namely that they approach zero for small strains. Since the damping ratio always is greater than zero, these models are unreliable at small strains.

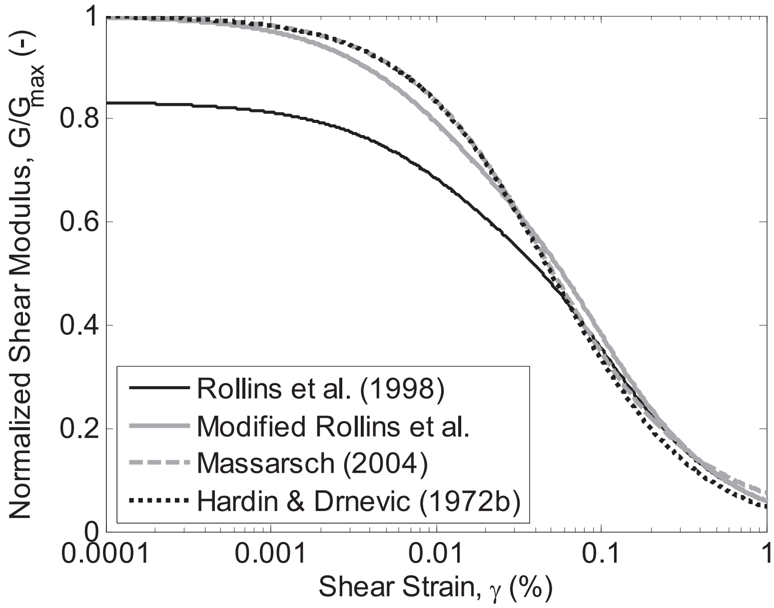


Figure 15. Strain-softening by different models.

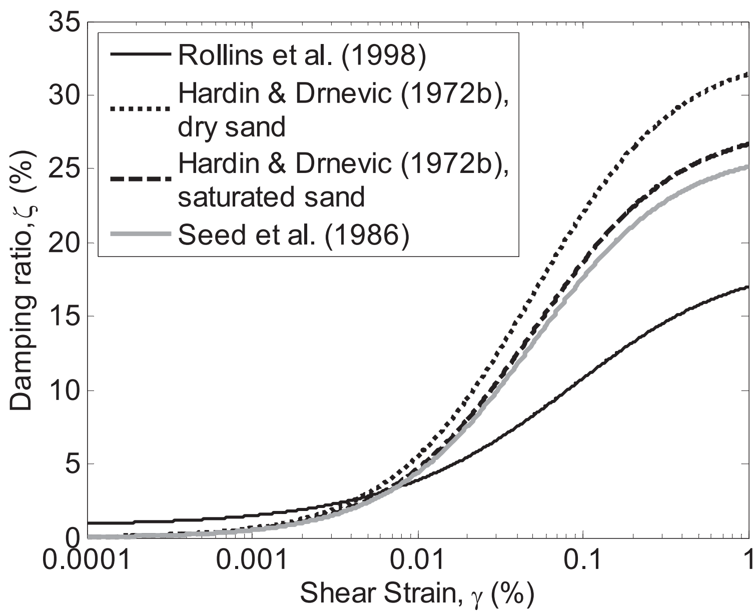


Figure 16. Comparison of damping ratio by different models.

4.1.4 Calculation of Foundation Response

The expressions given in Sections 4.1.1 and 4.1.2 can be used to calculate the frequency response for a vertically oscillating foundation on an elastic half-space. However, this is usually not sufficient for capturing the dynamic behavior of foundations on softening soil unless the strains are very small or the soil is highly plastic, as discussed above. During vibratory compaction, the case is normally the opposite, i.e. very large strains and non-plastic soil. This section describes a simple method to incorporate strain-softening behavior into the calculation of frequency response, proposed in Paper II. The procedure is here explained in relation to the small-scale tests conducted with rotating mass oscillators described in Chapter 2 but is equally applicable to oscillations of other types of foundations under different dynamic load. Paper III presents the method extended to a 2DOF and the tests with electro-dynamic oscillator.

The first step is to determine the uncorrected displacement amplitude frequency response. This is done by estimating the shear wave speed, c_S, and the mass density of the soil. The small-strain shear modulus can then be calculated by Equation 28.

$$G_{max} = \rho c_S^2 \tag{28}$$

Note that the shear wave speed in Equation 28 represents that at small strain and that it will decrease at larger strains. The small-strain shear wave speed can be measured by, for example, seismic tests. Alternatively, the small-strain shear modulus can be estimated directly by Equation 20. Determining Poisson's ratio of the soil and knowing the radius and mass of the foundation, the spring stiffness and damping ratio can be calculated by Equations 14-16. Depending on the relative size of the calculated apparent mass, it may be neglected and the mass of the foundation can be adopted as the total mass. Since the stiffness varies with frequency, each point on the curve will have a different natural frequency. The natural and dimensionless frequencies are calculated by Equations 6 and 7. Normally, the eccentric moment of the oscillator, $m_e e$, is known and the force amplitude can thus be calculated by Equation 10 for the frequency range of interest. The uncorrected frequency response for displacement amplitude is then obtained by Equation 8.

The next step is to calculate the shear strain in the soil. Each point in the response diagram represents a value of vertical displacement amplitude. This must first be converted to compressive strain and then to shear strain. Since a single value of strain is necessary for each frequency, strain must be assumed to be evenly distributed down to a certain depth. However, strain is assumed to approximately follow the Boussinesq distribution. To obtain a crude but representative value of the influence depth, the area under the Boussinesq stress-versus-depth-curve is simplified to a rectangular distribution corresponding to the maximum Boussinesq stress, i.e. that directly below the center of the plate. This gives the length of the strained element, L_ε. The compressive strain, ε, can then be calculated for each frequency by Equation 29 by using the displacement given by the response diagram.

$$\varepsilon = \frac{u_0}{L_\varepsilon} \tag{29}$$

By assuming axisymmetric conditions, the shear strain is calculated by Equation 30 (Atkinson & Bransby 1978).

$$\gamma = \frac{2}{3}\varepsilon(1+\nu) \tag{30}$$

After the shear strain has been obtained, new strain-dependent values of the shear modulus and damping ratio can be calculated for every frequency by a suitable formulation. In this study, Equation

27 was applied for shear modulus and Equation 22 for damping ratio. The maximum damping ratio was assumed to be 33 %, based on Equation 23. Since the calculation method is unable to deal with geometric damping and only material damping, the highest value for damping ratio is chosen.

The new shear modulus and damping ratio are then used, applying the same procedure, to calculate the frequency response for displacement, compressive strain and shear strain. The new shear strain again yields new values of the shear modulus and damping ratio and the process is repeated. This is iterated until the response diagrams converge with sufficiently small variations between iterations, for each value of frequency. The final displacement function then gives the frequency response for velocity amplitude v_0, acceleration amplitude a_0 and force amplitude F_0 by Equations 31 to 33.

$$v_0 = \omega u_0 \tag{31}$$

$$a_0 = \omega^2 u_0 \tag{32}$$

$$F_0 = k u_0 \tag{33}$$

4.2 Frequency Response in Small-Scale Tests

The equivalent-linear method was used to calculate the frequency response of dynamic parameters in the small-scale tests and compared to the measured values. The method was introduced in Paper II and applied to the tests using rotating-mass oscillators. The results showed that the equivalent-linear approach successfully can be applied to calculate dynamic properties, such as acceleration and force, even at very large strain. The calculations and measurements presented in Paper II illustrate the significant difference that is obtained in the response when strain-softening is taken into account and highlight the importance of soil nonlinearity. In Paper V, the same calculation method was applied to small-scale tests of varying force ratio. The acceleration and force response could be predicted with a reasonable accuracy for lower force ratios. At high dynamic force, however, there was a significant discrepancy between the measured and calculated response. This was attributed to higher vibration modes affecting the dynamic measurements and thus not to the calculation method. The resonant frequency could be estimated with a reasonable accuracy for all force ratios. Two main concerns about the calculation procedure are the crude representation of strain as constant down to a certain depth and the inability of the method to take geometric damping into account. However, the results suggest that the response can be predicted with a sufficient accuracy regardless of these simplifications.

In Paper III, the method was extended to a 2DOF and used to calculate the frequency response in the small-scale tests using the electro-dynamic oscillator. Here, the strain level was much lower than with the rotating mass oscillators. The calculated response matched the measured response very accurately.

4.3 Frequency Response in Full-Scale Tests

Calculating the response of a vibratory roller is more complex for several reasons. The cylindrical drum creates a non-uniform stress distribution between the drum and the soil and there is no clear definable contact width. Furthermore, the roller is in constant motion which can cause a deviation from the steady-state assumption. A preliminary calculation has been conducted with an assumed shear wave speed of 400 m/s and an assumed contact width of 100 mm and compared to the sweep test. The resulting dynamic displacement of the drum is shown in Figure 17. The absolute displacement cannot be fully captured but it seems the method can predict the resonant frequency. However, the calculated displacement shown below is based on assumed values and is thus preliminary. With a measured stiffness of the soil and more sophisticated determination of the contact area and stress distribution, the results will be more reliable. If the resonant frequency can be predicted, the optimum compaction frequency can also be predicted since it is found to be slightly above resonance, which implies that a suitable fixed-frequency roller can be chosen by calculating the response using soil and roller data. The need to experimentally determine the resonant frequency in the field is then eliminated.

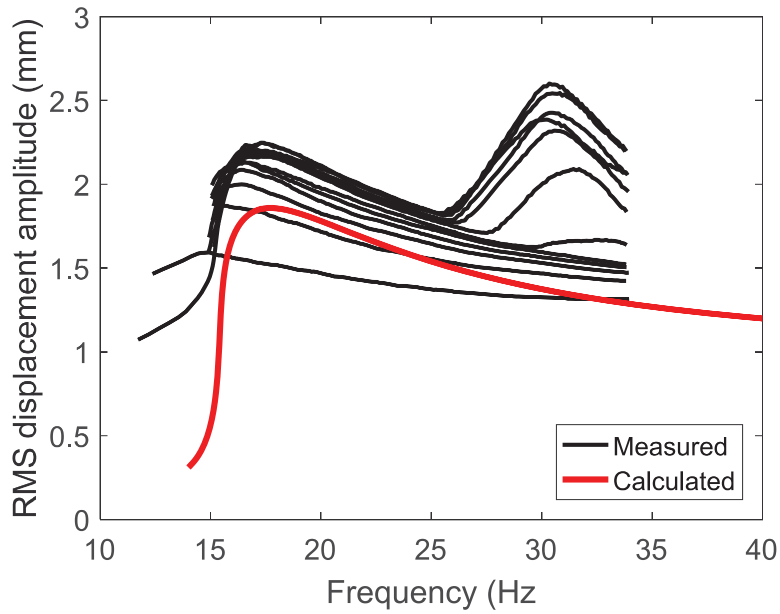


Figure 17. Measured and preliminary calculated dynamic displacement of the drum in the full-scale tests.

5 SUMMARY OF APPENDED PAPERS

5.1 Paper I

Small-Scale Testing of Frequency-Dependent Compaction of Sand Using a Vertically Vibrating Plate

Carl Wersäll and Stefan Larsson

Published in ASTM Geotechnical Testing Journal 2013:36(3)

The paper presents results from 85 small-scale tests that were conducted using a vertical electro-dynamic oscillator, connected to a plate and placed on a sand bed. Frequency was adjusted continuously to assess its influence on compaction of the underlying sand. The results showed that the rate of compaction with this type of compactor is significantly magnified at, and close to, the resonant frequency. The results indicated that velocity amplitude is a crucial quantity in obtaining sufficient compaction in the used test setup. While large velocity amplitude gave rise to a large degree of compaction, it also caused significant soil displacement and heave. Tests showed that compaction is closely related to strain-softening since the strain above which moduli start to decrease coincides with the strain required for compaction of the soil.

5.2 Paper II

Frequency Variable Surface Compaction of Sand Using Rotating Mass Oscillators

Carl Wersäll, Stefan Larsson, Nils Rydén and Ingmar Nordfelt

Published in Geotechnical Testing Journal 2015:38(2)

The objective of this paper is to study the influence of frequency in compaction tests using rotating mass oscillators. Results from 105 small-scale tests, conducted using a vertically oscillating plate, are presented. The soil underlying the plate was dry sand, or sand close to the optimum water content. The results showed that there is a resonant amplification, providing slightly higher degree of compaction. Most effective compaction is obtained at very high frequencies, but from an energy perspective, the optimum frequency is slightly above resonance. The paper discusses the implications for roller compaction and suggests that a lower frequency than what is applied today in practice may prove more efficient. An equivalent-linear iterative method for calculating dynamic response of the plate, incorporating strain-dependent properties of the soil, is also presented. The calculated frequency response agrees well with measured quantities.

5.3 Paper III

Dynamic Response of Vertically Oscillating Foundations at Large Strain

Carl Wersäll, Stefan Larsson and Anders Bodare

Published in Proceedings of the 14th International Conference of the International Association for Computer Methods and Advances in Geomechanics, Kyoto, Japan, 22-25 September 2014

The equivalent-linear method developed in Paper II is here extended from a SDOF to a 2DOF system. The tests described in Paper I, using an electro-dynamic oscillator, can be represented with such a model and the results from those tests are thus used as verification. Calculations of dynamic displacement amplitude are compared to measured displacement and the results agree well. Since the strain amplitude was quite low, while it was very high in the previous study, the results show that the method can be applied to a wide range strain levels.

5.4 Paper IV

Soil Compaction by Vibratory Roller with Variable Frequency

Carl Wersäll, Ingmar Nordfelt and Stefan Larsson

Submitted to Géotechnique February 2016

The paper describes full-scale tests conducted with a 12100 kg vibratory soil compaction roller, modified to operate under a variable frequency. Nine fixed frequencies in the range 15-35 Hz were applied and one sweep test where the frequency was varied linearly during compaction. The material consisted of crushed gravel and, apart from roller-integrated measurements, settlement was measured by laser levelling and density by nuclear density gauge. The results revealed that the resonant frequency was 17 Hz, while the optimum compaction frequency was 18 Hz, i.e. slightly above resonance. Amplification around the resonant frequency, in combination with a reduction in compaction at high frequencies due to double jump of the drum, results in a near constant compaction efficiency above resonance. Since the standard operating frequency of this particular roller is 31 Hz, a frequency reduction of more than 10 Hz is feasible without affecting the compaction result. A lower frequency implies a significant reduction in fuel consumption, environmental impact and machine wear. The study also showed that loosening of top soil can be avoided by compacting at a lower frequency, thus eliminating the need for subsequent static passes.

5.5 Paper V

Influence of Force Ratio and Frequency on Vibratory Surface Compaction

Carl Wersäll and Stefan Larsson

In addition to the tests presented in Paper II, 55 further small-scale tests were conducted and compared to the previous results. The purpose was to study the influence of force ratio, i.e. the ratio of dynamic force and static weight, on the frequency-dependent compaction behavior. By keeping the same oscillators and removing static weight, the force ratio could be increased. When reducing the system mass, the resonant frequency increases. However, the frequency-dependent behavior seems to be similar for the three tested force ratios and the optimum compaction frequency can be assumed to be slightly above resonance in all cases. The results imply that rollers with other properties than the one used in the full-scale most likely will provide the most energy-efficient compaction at frequencies close to resonance.

6 CONCLUSIONS

This thesis presents results from frequency-dependent compaction in small-scale tests using a vertically vibrating plate and full-scale tests using a vibratory soil compaction roller. In the small-scale tests, two test setups were manufactured, the first using an electro-dynamic oscillator and the second utilizing two rotating mass oscillators. The frequency response was calculated by combining theory for vibrating foundations on elastic half-space with an iterative equivalent-linear procedure for estimating strain-dependent properties of soil. For discussion and detailed conclusions from each study, reference is made to the individual papers appended to this this thesis. The main conclusions of all studies incorporated in this thesis are listed below:

- Soil compaction by a vibratory plate is frequency-dependent, providing an amplified degree of compaction close to the resonant frequency.
- The resonant amplification is very significant when the dynamic force is very low compared to the static weight. When the dynamic force is greater than the static weight, amplification is more modest due to different loading conditions and large strain causing high damping and stiffness reduction in the soil.
- When the dynamic force is varied slightly, but still greater than the static weight, the frequency-dependent compaction behavior is similar. However, the resonant frequency changes.
- The small-scale tests using an electro-dynamic oscillator gave quite a large amount of soil displacement and heave while these were small in the rotating mass oscillator tests.
- No dynamic quantity is solely governing for the degree of compaction.
- The water content of the soil has no apparent effect on the compaction frequency dependence. It does, however, have a positive effect on the absolute degree of compaction.
- Compaction is closely related to strain-softening. The strain level, above which the stiffness of the soil starts to decrease, coincides with the strain required to obtain rearrangement of soil particles.
- The optimum compaction frequency for a vibratory roller is slightly above the coupled roller-soil resonant frequency when considering the average densification of the compacted layer.
- The optimum compaction frequency also provides an even density distribution throughout the layer. At higher frequencies, the bottom of the layer is more extensively densified while the top part is loosened. Avoiding loosening of the top soil layer can limit the need for subsequent static passes.
- The roller used in the full-scale tests had a standard operating frequency which was 13 Hz above the optimum frequency of the tests, implying that compaction can be made more energy-efficient.
- There is no direct correlation between the centrifugal force of the roller and compaction efficiency and it should thus not be used as a measure of compaction capacity.
- The proposed method to calculate frequency response captures, in most cases, the dynamic behavior but need to be verified for roller compaction using measured parameters. If successful, it can be used to estimate the resonant frequency.

In summary, the combination of small-scale and full-scale tests provided valuable insights regarding surface compaction of granular soil. The first tests with an electro-dynamic oscillator showed that resonance is immensely important for a vibrating plate when the dynamic force is very low, where the amount compaction below or above resonance becomes insignificant in comparison. The settlement velocities in all tests using the electro-dynamic oscillator are presented in Figure 18. In practical applications of surface soil compaction, however, the dynamic load is normally generated by rotating eccentric masses. Here, the dynamic force normally exceeds the static weight and the equipment can lose contact with the soil. The loading conditions are thus very different and soil nonlinearity also has a dramatic effect on the response of the soil. Furthermore, rotating masses produce a centrifugal force that increases significantly with frequency. The small-scale tests using rotating mass oscillators imitated these conditions in a laboratory environment. The results, some of which are shown in Figure 19, revealed that the best compaction is obtained at very high frequency but suggested that the optimum frequency from an energy perspective is close to the resonant frequency. This was confirmed in the full-scale tests, with the exception of a very high frequency causing chaotic motion of the drum and thus not suitable for compaction. The optimum compaction frequency is slightly above resonance, as can be observed in Figure 20. The roller used for the tests described herein had a standard frequency which was 13 Hz higher than the optimum and it can be assumed that compacting at frequencies far above optimum is common. Reducing the frequency has a significant effect on energy consumption and machine wear and it is recommended that a procedure for practical resonance compaction is developed. The resonant frequency can be obtained by roller-integrated measurement. Alternatively, it can be calculated and the equivalent-linear method proposed in this thesis seems promising for predicting the resonant frequency of the roller-soil system.

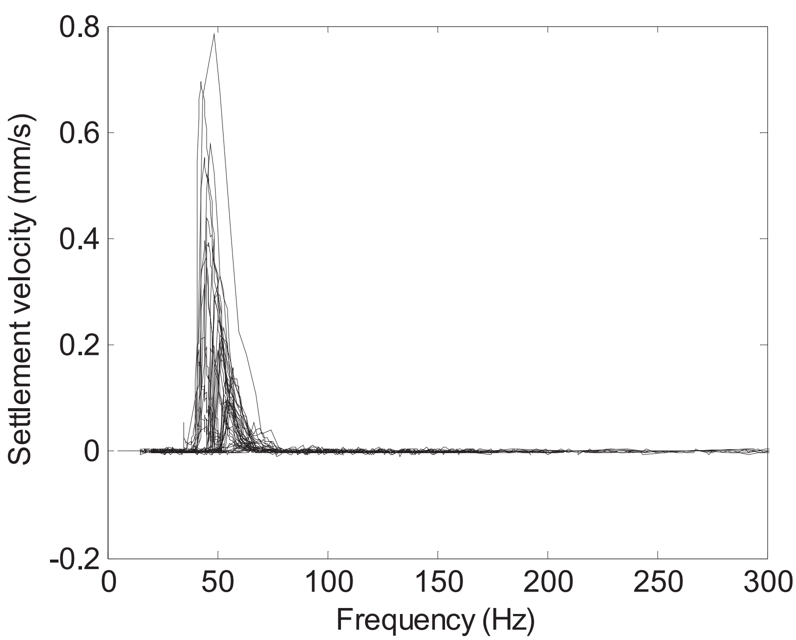


Figure 18. Settlement velocity in tests using electro-dynamic oscillator. Modified after Paper I.

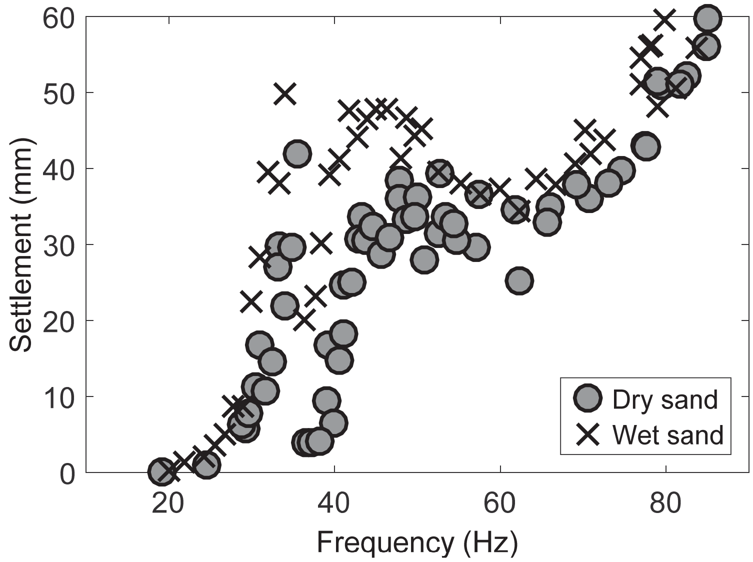


Figure 19. Total settlement in tests using rotating mass oscillators. From Paper V.

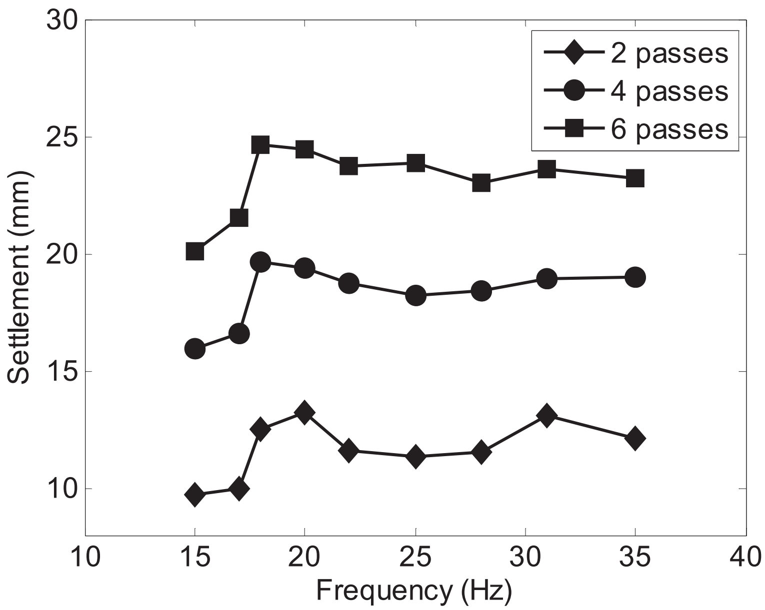


Figure 20. Frequency-dependent settlement in full-scale tests.

REFERENCES

Adam, D., 1996., *Flächendeckende Dynamische Verdichtungskontrolle (FDVK) mit Vibrationswalzen* [Continuous Compaction Control with Vibratory Rollers], PhD thesis, University of Vienna, Vienna, Austria, 267 p., (in German).

Amir-Faryar, B., Aggour, M. S. and McCuen, R. H., 2016, “Universal Model Forms for Predicting the Shear Modulus and Material Damping of Soils,” *Geomechanics and Geoengineering: An International Journal*, in press.

Anderegg, R. and Kaufmann, K., 2004, “Intelligent Compaction with Vibratory Rollers,” *Transportation Research Record. 1868*, Transportation Research Board, Washington, D.C., pp. 124–134.

Assimaki, D., Kausel, E., and Whittle, A., 2000, ”Model for Dynamic Shear Modulus and Damping for Granular Soils.” *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 126, No. 10, pp. 859–869.

Atkinson, J. H., Bransby, P. L., 1978, *The Mechanics of Soils: An Introduction to Critical State Soil Mechanics*, McGraw-Hill, London, 375 p.

Baidya, D. K. and Murali Krishna, G., 2001, “Investigation of Resonant Frequency and Amplitude of Vibrating Footing Resting on a Layered Soil System,” *Geotechnical Testing Journal*, Vol. 24, No. 4, pp. 409–417.

Bernhard, R. K., 1952, “Static and Dynamic Soil Compaction,” *Highway Research Board Proceedings*, Vol. 31, pp. 563–592.

Converse, F. J., 1953, “Compaction of Sand at Resonant Frequency,” *ASTM Special Technical Publication*, No. 156, pp. 124–137.

D’Appolonia, D. J., Whitman, R. V. and D’Appolonia, E., 1969, “Sand Compaction with Vibratory Rollers,” *Journal of the Soil Mechanics and Foundations Division*, Vol. 95, No. 1, pp. 263–284.

Dobry, R. and Whitman, R. V., 1973, “Compaction of Sand on a Vertically Vibrating Table,” *Evaluation of Relative Density and Its Role in Geotechnical Projects Involving Cohesionless Soils*, ASTM Special Technical Publication, No. 523, pp. 156–170.

Facas, N. W., Rinehart, R. V. and Mooney, M. A., 2011, “Development and Evaluation of Relative Compaction Specifications Using Roller-Based Measurements,” *Geotechnical Testing Journal*, Vol. 34, No. 6, pp. 634–642.

Forssblad, L., 1965, “Investigations of Soil Compaction by Vibration,” *Acta Polytechnica Scandinavica*, No. 34.

Forssblad, L., 1980, “Compaction Meter on Vibrating Rollers for Improved Compaction Control,” *Proceedings of International Conference on Compaction*, Vol. 2, Paris, France, pp. 541–546.

Forssblad, L., 1981, *Vibratory Soil and Rock Fill Compaction*, Dynapac Maskin AB, Stockholm, Sweden, 175p.

Gazetas G., 1983, "Analysis of Machine Foundation Vibrations: State-of-the-Art," *International Journal of Soil Dynamics and Earthquake Engineering*, Vol. 2, No. 1, pp. 2-43.

Hall, J. R., Jr., 1967, "Coupled Rocking and Sliding Oscillations of Rigid Circular Footings," *Proceedings of the International Symposium on Wave Propagation and Dynamic Properties of Earth Materials*, Albuquerque, New Mexico, pp. 139-148.

Hardin, B. O., 1978, "The Nature of Stress-Strain Behavior of Soils," *Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference on Earthquake Engineering*, Vol. 1, Pasadena, California, pp. 3-90.

Hardin, B. O. and Drnevich, V. P., 1972a, "Shear Modulus and Damping in Soils: Design Equations and Curves," *Journal of the Soil Mechanics and Foundations Division*, Vol. 98, No. 7, pp. 667–692.

Hardin, B. O. and Drnevich, V. P., 1972b, "Shear Modulus and Damping in Soils: Measurement and Parameter Effects," *Journal of the Soil Mechanics and Foundations Division*, Vol. 98, No. 6, pp. 603-624.

Hertwig, A., 1936, "Means for Consolidating the Ground," US Patent 2039078.

Johnson, A. W. and Sallberg, J. R., 1960, *Factors that Influence Field Compaction of Soils*, Highway Research Board, Bulletin No. 272, 206 p.

Kausel, E. and Assimaki, D., 2002, "Seismic Simulation of Inelastic Soils via Frequency-Dependent Moduli and Damping." *Journal of Engineering Mechanics*, Vol. 128, No. 1, pp. 34–47.

Kondner, R. L., 1963a, *A Hyperbolic Stress-strain Formulation for Sands*, Northwestern University, 60 p.

Kondner, R. L., 1963b, "A Hyperbolic Stress-Strain Response: Cohesive Soils," *Journal of the Soil Mechanics and Foundations Division*, Vol. 89, No. 1, pp. 115-143.

Lorenz, H., 1960, *Grundbau-Dynamik* [Foundation Dynamics], Springer-Verlag, Berlin, 308 p., (in German).

Lysmer, J. and Richart, F. E., Jr., 1966, "Dynamic Response of Footings to Vertical Loading," *Journal of the Soil Mechanics and Foundations Division*, Vol. 92, No. 1, pp. 65–91.

Mandal, A., Baidya, D. K. and Roy, D., 2012, "Dynamic Response of the Foundations Resting on a Two-layered Soil Underlain by a Rigid Layer," *Journal of Geotechnical and Geological Engineering*, Vol. 30, No. 4, pp. 775-786.

Masing, G., 1926, "Eigenspannungen und Verfestigung beim Messing," *Proceedings of the second International, Congress on Applied Mechanics*, pp. 332-335, (in German).

Massarsch, K. R., 2004, “Deformation Properties of Fine-Grained Soils from Seismic Tests,” Keynote lecture, *Proceedings of the Second International Conference on Site Characterization, ISC’2*, Porto, Portugal, pp. 133–146.

Mooney, M. A. and Rinehart, R. V., 2007, “Field Monitoring of Roller Vibration During Compaction of Subgrade Soil,” *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 133, No. 3, pp. 257-265.

Mooney, M. A. and Rinehart, R. V., 2009, “In Situ Soil Response to Vibratory Loading and Its Relationship to Roller-Measured Soil Stiffness,” *Journal of Geotechnical and Geological Engineering*, Vol. 135, No. 8, pp. 1022–1031.

Novak, M., 1970, “Prediction of Footing Vibrations,” *Journal of the Soil Mechanics and Foundations Division*, Vol. 96, No. 3, pp. 837-861.

Rad, N. S. and Tumay, M. T., 1987, “Factors Affecting Sand Specimen Preparation by Raining,” *Geotechnical Testing Journal*, Vol. 10, No. 1, pp. 31–37.

Richart, F. E., Jr., Hall, J. R., Jr. and Woods, R. D., 1970, *Vibrations of soils and foundations*, Prentice-Hall, Englewood Cliffs, New Jersey, 414 p.

Rollins, K. M., Evans, M. D., Diehl, N. B. and Daily, W. D. III, 1998, ”Shear Modulus and Damping Relationships for Gravels.” *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 124, No. 5, pp. 396–405.

Schnabel, P. B., Lysmer, J. and Seed, H. B., 1972, *SHAKE: a Computer Program for Earthquake Response Analysis of Horizontally Layered Sites*, Earthquake Engineering Research Center, University of California, Berkeley, 92 p.

Seed, H. B., Wong, R. T., Idriss, I. M. and Tokimatsu, K., 1986, ”Moduli and Damping Factors for Dynamic Analyses of Cohesionless Soils,” *Journal of Geotechnical Engineering*, Vol. 112, No. 11, pp. 1016–1032.

Stokoe, K. H., II, Darendeli, M. B., Andrus, R. D., and Brown, L. T., 1999, “Dynamic soil properties: Laboratory, field and correlation Studies,” *Proceedings of the Second International Conference on Earthquake Geotechnical Engineering*, Vol. 3, Lisbon, Portugal, pp. 811–845.

Susante, P. J. and Mooney, M. A., 2008, “Capturing Nonlinear Vibratory Roller Compactor Behavior through Lumped Parameter Modeling,” *Journal of Engineering Mechanics*, Vol. 134, No. 8, pp. 684-693.

Tatsuoka, F., Masuda, T., Siddiquee, M., and Koseki, J., 2003, ”Modeling the Stress-Strain Relations of Sand in Cyclic Plane Strain Loading.” *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 129, No. 5, pp. 450–467.

Thurner, H. and Sandström, Å., 1980, "A New Device for Instant Compaction Control," *Proceedings of International Conference on Compaction*, Vol. 2, Paris, France, pp. 611–614.

Tschebotarioff, G. P. and McAlpin, G. W., 1947, *Effect of Vibratory and Slow Repetitional Forces on the Bearing Properties of Soils*, Civil Aeronautics Administration, Technical Development Report No. 57, 70 p.

Vucetic, M. and Dobry, R., 1991, "Effect of Soil Plasticity on Cyclic Response," *Journal of Geotechnical Engineering*, Vol. 117, No. 1, pp. 89–107.

Wersäll, C. and Larsson, S., 2013, "Small-Scale Testing of Frequency-Dependent Compaction of Sand Using a Vertically Vibrating Plate," *Geotechnical Testing Journal*, Vol. 36, No. 3, pp. 1-10.

Wersäll, C., Larsson, S. and Bodare, A., 2014, "Dynamic Response of Vertically Oscillating Foundations at Large Strain," *Proceedings of the 14th International Conference of the International Association for Computer Methods and Advances in Geomechanics*, Kyoto, Japan, 22-25 September 2014, pp. 643-647.

Wersäll, C., Larsson, S., Rydén, N. and Nordfelt, I., 2015, "Frequency Variable Surface Compaction of Sand Using Rotating Mass Oscillators," *Geotechnical Testing Journal*, Vol. 38, No. 2, 2015, pp. 1–10.

Wersäll, C., Nordfelt, I. and Larsson, S., "Soil Compaction by Vibratory Roller with Variable Frequency," Submitted to Géotechnique, February 2016.

Wersäll, C. and Larsson, S., "Influence of Force Ratio and Frequency on Vibratory Surface Compaction," Submitted to Geotechnics for Sustainable Infrastructure Development – Geotec Hanoi 2016.

Yoo, T-S. and Selig, E. T., 1979, "Dynamics of Vibratory-Roller Compaction," *Journal of the Geotechnical Engineering Division*, Vol. 105, No. 10, pp. 1211–1231.

Zhang, J., Andrus, R., and Juang, C., 2005, "Normalized Shear Modulus and Material Damping Ratio Relationships." *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 131, No. 4, pp. 453–464.